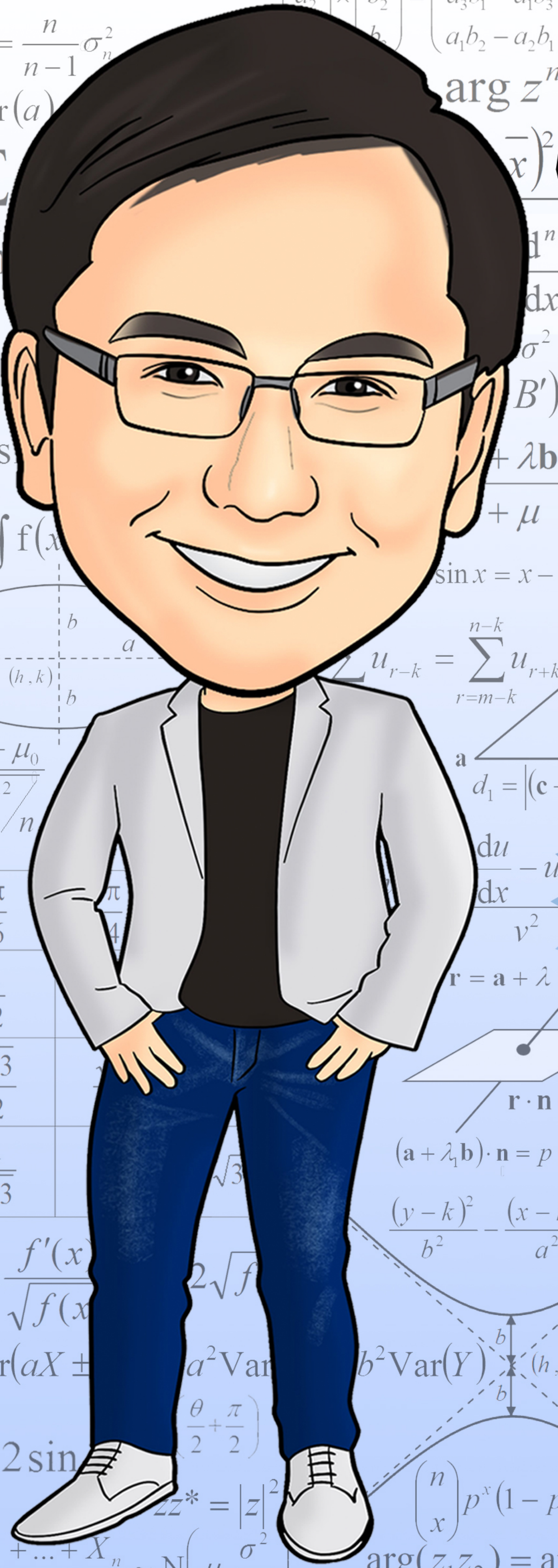
A-LEVEL MATHS

MADE EASY

Your Quick Reference To Success

Jackie Lee

Singapore Supreme H2 Mathematics Specialist



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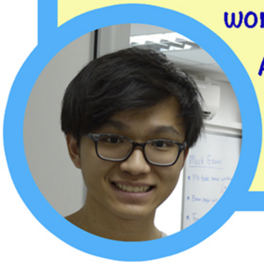
Mr Jackie Lee was formerly teaching Mathematics in several good schools, including **Raffles Junior College** and **Anglo-Chinese School (Independent)**. He was also the team leader of preparing students for Mathematics competitions, such as **Singapore Mathematics Olympiad (SMO)**. Since the introduction of graphing calculator in 2007, Mr Lee was also the author of the best seller **"Practical Guide To GCE A-Level H2 Mathematics (GC Approach)"**. He obtained his Master degree in **National University of Singapore (NUS)** as well as the **Postgraduate Diploma in Education (PGDE)** from **National Institute of Education (NIE)** with Distinction in teaching Mathematics.

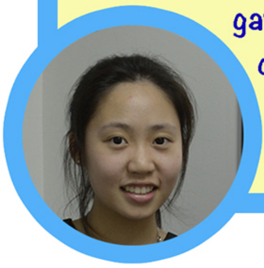
Prior to that, Mr Lee was also the recipient of two scholarships awarded by **Ministry of Education (MOE)** in 1992 and **Economic Development Board (EDB)** in 2001.

Mr Lee is certainly very passionate in his teaching. Although I joined really late, he clarified all my queries regarding concepts in various topics and gave me a list of commonly tested questions which granted me greater confidence before the exam. All his dedicated works allowed me to score A for math in A-level. Thank you Mr Lee!

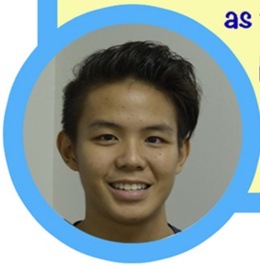


His unique style of lecturing and excellent selection of teaching examples as well as practice questions were extremely helpful. Before I attended Mr Lee's class, I had troubles with my Mathematics. However, after attending his classes plus countless consultations, my grades started rising and I managed to clinch A in A level! Thank you Mr Lee!

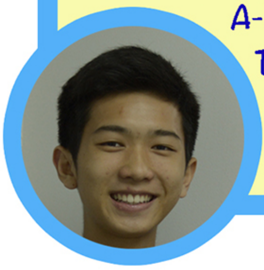
I managed to improve from E in Prelim to A in A-level within 3 months. The revision package was useful and the drilling of the topics in class helped me understand and know the question types better. The different ways of answering and approaching the questions gave me great insights and helped me change the way I think and answer. Thanks Mr Lee!



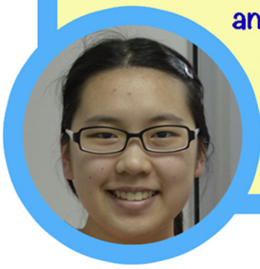
Mr Lee showed care and concern during his lessons, having planned each session with progressive yet challenging questions. This helped me gain a better understanding of Math in general and helped me grasp concepts better. I am glad to have scored A in A-level as this is the best way, in my opinion, to pay back his hard work towards me. Thanks Mr Lee!



Hi Mr Lee, Thank you so much for your effective teaching methods that made math a lot easier for me to understand! The mini practice booklets with the checklists were also really helpful in revision. I was surprised I jumped 4 grades from D in prelims to A in A-level. The extra practice really paid off. Thank you Mr Lee!



In school, the results I got for Math were from S to D. Though I joined Mr Lee's tuition very late with only 2 months left, I was still able to benefit from his guidance immensely and finally scored an A grade in A Levels. Thanks Mr Lee for helping me with all the practices and detailed explanations. He is a great Math tutor in Singapore.



I attended Mr Lee's lessons when my maths concept was still shaky. After attending his crash course on vectors, I decided to sign up for his classes. Through these classes, he taught me various methods and tricks to answer different question types. My grade improved from E in my common test to A at A Level! Thank you Mr Lee for being so patient and helpful.

I'm very thankful to Mr Lee's guidance of helping me get an A in the A Levels. I was not doing well in math before I joined but he made me believe that I could reach my goals. The materials given were enough to give me sufficient exposure to different question types. And I've gained more confidence in math in which I was severely lacking before joining his class.

<WORDS OF WISDOM>

Everybody is a genius.
But if you judge a **fish** by its ability
to **climb** a tree, it will live its
whole life **believing** that it is stupid.

– Albert Einstein –

Sometimes when you **innovate**,
you **make mistakes**.
It is best to **admit** them **quickly**, and
get on with **improving** your other innovations.

– Steve Jobs –

Today is **hard**, tomorrow will be **worse**,
but the **day after tomorrow** will be **sunshine**.

– Jack Ma –

Success is **not final**, Failure is **not fatal**,
It is the **courage** to continue that counts.

– Winston Churchill –

Content

Algebraic Identities

$a^2+b^2=(a+b)^2-2ab$	$a^2-b^2=(a-b)(a+b)$
$a^3+b^3=(a+b)(a^2-ab+b^2)$	$a^3-b^3=(a-b)(a^2+ab+b^2)$

Inequalities: Basic Rules

If	then
$a>b$ and $b>c$	$a>b>c$
$a>b$	$a\pm c>b\pm c$
$a>b>0$	$\frac{1}{a}<\frac{1}{b}$
$a>b$ and $c>0$	$ac>bc$ and $\frac{a}{c}>\frac{b}{c}$
$a>b$ and $c<0$	$ac<bc$ and $\frac{a}{c}<\frac{b}{c}$
$ab>0$	$a>0$ and $b>0$ or $a<0$ and $b<0$
$ab<0$	$a>0$ and $b<0$ or $a<0$ and $b>0$
$0<a<1$	$a^2<a$ and $a<\sqrt{a}$
$a>1$	$a^2>a$ and $a>\sqrt{a}$

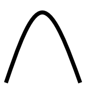
Inequalities: involving Modulus

Given $a\geq 0$,	$\lvert x\rvert\geq a\Rightarrow x\geq a$ or $x\leq -a$	$\lvert x\rvert\leq a\Rightarrow -a\leq x\leq a$
Given $x,y\in\Re$,	$\lvert x\rvert\leq\lvert y\rvert\Rightarrow x^2\leq y^2$	$\lvert x\rvert\geq\lvert y\rvert\Rightarrow x^2\geq y^2$

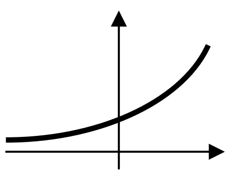
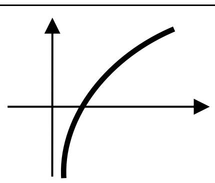
Quadratic Equation

$ax^2+bx+c=0\Rightarrow$

b^2-4ac	>0	$=0$	<0
Nature of Roots	2 Distinct	1 Repeated	No Real

$a>0$	$a<0$	x-intercept(s)	y-intercept(s)	Turning Point
		$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	c	$\left(-\frac{b}{2a},c-\frac{b^2}{4a}\right)$

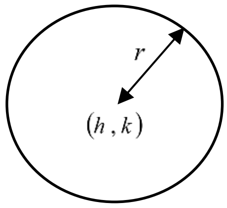
Exponential & Logarithmic Graphs

Exponential		**Logarithmic**	
	$x\in R$ $y>0$ $(0,1)$ $x\to-\infty,\ y\to 0^+$		$x>0$ $y\in R$ $(1,0)$ $x\to 0^+,\ y\to-\infty$

Graphing Techniques: Conics Graphs

Circle

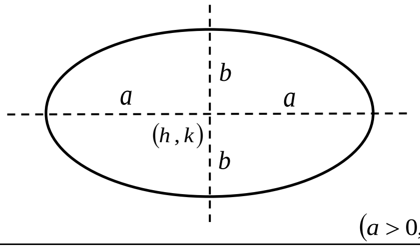
$$(x-h)^2+(y-k)^2=r^2$$



$(r>0)$

Ellipse

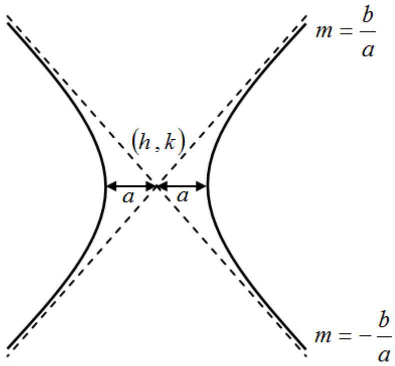
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$



$(a>0, b>0)$

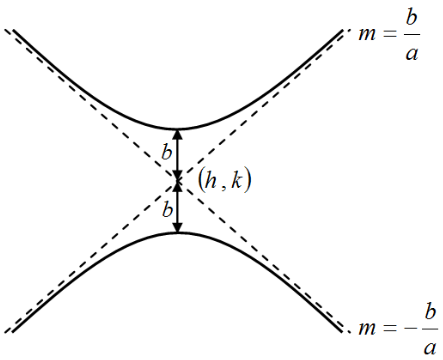
Hyperbola

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$



$(a>0, b>0)$

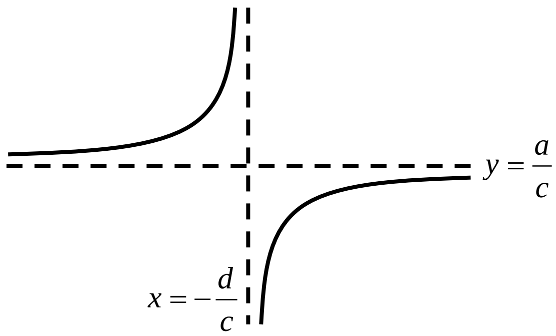
$$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$$

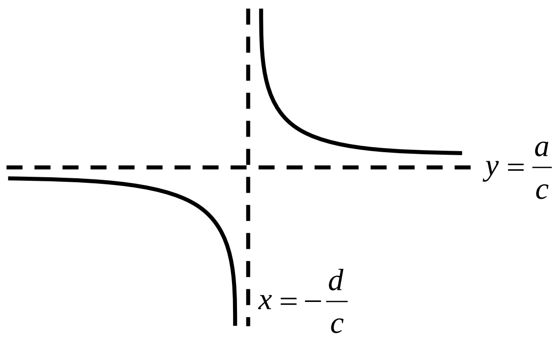


$(a>0, b>0)$

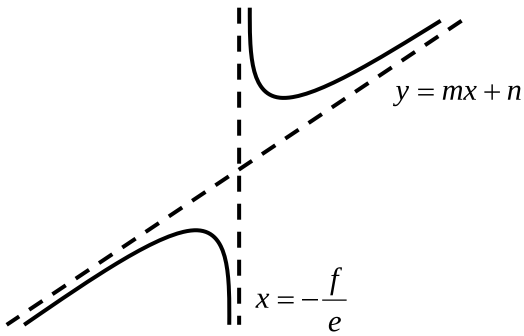
Graphing Techniques: Graphs of Rational Fraction

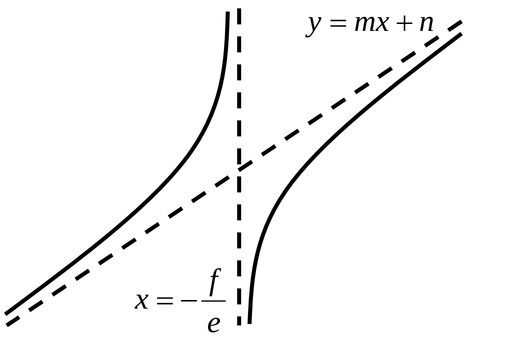
$$y=\frac{ax+b}{cx+d}=\frac{a}{c}+\frac{k}{cx+d}$$





$$y=\frac{ax^2+bx+c}{ex+f}=mx+n+\frac{k}{ex+f}$$





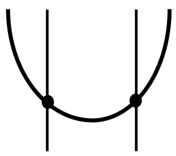
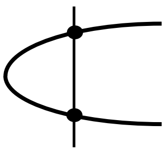
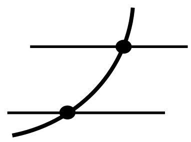
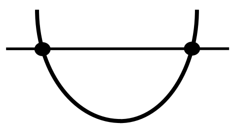
Graphical Transformations: Translation, Scaling & Reflection

$\mathrm{f}(x)$	**Horizontal**		**Vertical**	
Translation	$\mathrm{f}(x+k)$	k units	$\mathrm{f}(x)+k$	k units
Scaling	$\mathrm{f}(kx)$	$\times\frac{1}{k}$	$k\mathrm{f}(x)$	$\times k$
Reflection	$\mathrm{f}(-x)$	y	$-\mathrm{f}(x)$	x

Graphical Transformations: Composite Horizontal Transformations

$\mathrm{f}(x)$	**Step 1**		**Step 2**	
Method 1	$\mathrm{f}(ax)$	$\times\frac{1}{a}$	$\mathrm{f}\left(a\left(x+\frac{b}{a}\right)\right)=\mathrm{f}(ax+b)$	$\frac{b}{a}$ units
Method 2	$\mathrm{f}(x+b)$	b units	$\mathrm{f}(ax+b)$	$\times\frac{1}{a}$

Functions: Vertical Line Testing & Horizontal Line Testing

Vertical Line Testing		**Horizontal Line Testing**	
To determine if a function can be defined.		To determine if a function is one-one.	
$x=k, k\in D_{\mathrm{f}}$ cuts $\mathrm{f}(x)$ exactly once.		$y=k, k\in R_{\mathrm{f}}$ cuts $\mathrm{f}(x)$ exactly once.	
			
Yes	**No**	**Yes**	**No**

Functions: Relationship between Original and its Inverse Function

(a,b) of $\mathrm{f}(x)\Leftrightarrow(b,a)$ of $\mathrm{f}^{-1}(x)\ \equiv\ \mathrm{f}(a)=b\Leftrightarrow a=\mathrm{f}^{-1}(b)$.
$y=x$ is the line of symmetry between $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$.
$\mathrm{f}(x)=\mathrm{f}^{-1}(x)=x$ occurs at the point of intersection(s) if existing.

Functions: Condition for Inverse / Composite Function to Exist

Inverse Function $\mathrm{f}^{-1}(x)$	**Composite Function** $\mathrm{fg}(x)$
$\mathrm{f}(x)$ is one-one. [Horizontal Line Testing]	$R_{\mathrm{g}}\subseteq D_{\mathrm{f}}$

Functions: Domain & Range of Inverse / Composite Function

	Inverse Function $f^{-1}(x)$	Composite Function $fg(x)$
Domain	$D_{f^{-1}} = R_f$	$D_{fg} = D_g$
Range	$R_{f^{-1}} = D_f$	$R_{fg} = R_f$ using $D_f \cap R_g$ as domain of f

Partial Fractions

MF26

Non-repeated linear factors: $\dfrac{px+q}{(ax+b)(cx+d)} = \dfrac{A}{(ax+b)} + \dfrac{B}{(cx+d)}$

Repeated linear factors: $\dfrac{px^2+qx+r}{(ax+b)(cx+d)^2} = \dfrac{A}{(ax+b)} + \dfrac{B}{(cx+d)} + \dfrac{C}{(cx+d)^2}$

Non-repeated quadratic factor: $\dfrac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \dfrac{A}{(ax+b)} + \dfrac{Bx+C}{(x^2+c^2)}$

Binomial Expansions: where n is a positive integer

MF26

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}ab^{n-1} + b^n \text{ where } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Binomial Expansions: where n is a fraction / negative integer

MF26

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x|<1)$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots \qquad (1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots$$

$$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots \qquad (1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots$$

Maclaurin's Expansion: Standard Results

MF26

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^n}{n!}f^{(n)}(0) + \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^r}{r!} + \ldots \qquad (\text{all } x)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \ldots \qquad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!} + \ldots \qquad (\text{all } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1} x^r}{r} + \ldots \qquad (-1 < x \le 1)$$

Maclaurin's Expansion: Small Angle Approximations

$\sin x \approx x$	$\cos x \approx 1 - \frac{1}{2}x^2$	$\tan x \approx x$

Note: x is in radian.

AP & GP: Standard Formulae

	AP	GP
T_n	$T_n = S_n - S_{n-1}$	
	$a + (n-1)d$	ar^{n-1}
S_n	$\frac{n}{2}[2a + (n-1)d]$ or $\frac{n}{2}(a + T_n)$	$\frac{a(r^n - 1)}{r-1}$ or $\frac{a(1 - r^n)}{1 - r}$
S_∞	-	$\frac{a}{1-r}$ $\quad |r| < 1$

AP & GP: Equation involving 3 Consecutive Terms

	AP	GP
If x, y and z are consecutive terms of AP / GP, then	$y - x = z - y$	$\frac{y}{x} = \frac{z}{y}$

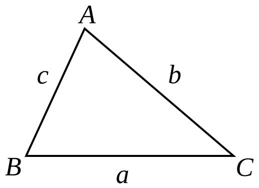
AP & GP: To Prove if the Sequence is AP / GP

	AP	GP
If the sequence is AP / GP, then	$T_{n+1} - T_n = \text{constant}$	$\frac{T_{n+1}}{T_n} = \text{constant}$

Summation: Standard Results involving Sigma Notation

$\sum_{r=m}^{n} a = (n - m + 1)a$	$\sum_{r=1}^{n} a^r = \frac{a(1 - a^n)}{1 - a}$	$\sum_{r=m}^{n} (au_r) = a\sum_{r=m}^{n} u_r$
$\sum_{r=m}^{n} (u_r \pm v_r) = \sum_{r=m}^{n} u_r \pm \sum_{r=m}^{n} v_r$	$\sum_{r=0}^{n} u_r = \sum_{r=1}^{n} u_r + u_0$	$\sum_{r=2}^{n} u_r = \sum_{r=1}^{n} u_r - u_1$
$\sum_{r=m}^{n} u_r = \sum_{r=1}^{n} u_r - \sum_{r=1}^{m-1} u_r$	$\sum_{r=m}^{\infty} u_r = S_\infty - \sum_{r=1}^{m-1} u_r$	$\sum_{r=m}^{n} u_r = \sum_{r=m+k}^{n+k} u_{r-k} = \sum_{r=m-k}^{n-k} u_{r+k}$
$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$	$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$

Trigonometry: Sine Rule & Cosine Rule



Sine Rule	Cosine Rule
$\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$	$a^2=b^2+c^2-2bc\cos A$
	$\cos A=\dfrac{b^2+c^2-a^2}{2bc}$

Trigonometry: Special Angle Table

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	How to remember?
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{0,1,2,3,4}}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{\sqrt{4,3,2,1,0}}{2}$
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$\frac{\sin\theta}{\cos\theta}$

Trigonometry: Basic Formulae

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ $\quad \sin^2\theta+\cos^2\theta=1$ $\quad \tan^2\theta+1=\sec^2\theta$ $\quad \cot^2\theta+1=\operatorname{cosec}^2\theta$

Trigonometry: Principal Values

MF26

$-\frac{1}{2}\pi\le\sin^{-1}x\le\frac{1}{2}\pi\quad(|x|\le 1)$ $\quad 0\le\cos^{-1}x\le\pi\quad(|x|\le 1)$ $\quad -\frac{1}{2}\pi<\tan^{-1}x<\frac{1}{2}\pi$

Trigonometry: Compound Angle Formulae

MF26

$\sin(A\pm B)\equiv\sin A\cos B\pm\cos A\sin B$

$\cos(A\pm B)\equiv\cos A\cos B\mp\sin A\sin B$

$\tan(A\pm B)\equiv\dfrac{\tan A\pm\tan B}{1\mp\tan A\tan B}$

Trigonometry: Compound Angle involving 90,180,270,360 degrees

$\sin(90^\circ-\theta)\equiv\cos\theta$	$\cos(90^\circ-\theta)\equiv\sin\theta$	$\tan(90^\circ-\theta)\equiv 1/\tan\theta$
$\sin(90^\circ+\theta)\equiv\cos\theta$	$\cos(90^\circ+\theta)\equiv-\sin\theta$	$\tan(90^\circ+\theta)\equiv -1/\tan\theta$
$\sin(180^\circ-\theta)\equiv\sin\theta$	$\cos(180^\circ-\theta)\equiv-\cos\theta$	$\tan(180^\circ-\theta)\equiv-\tan\theta$
$\sin(180^\circ+\theta)\equiv-\sin\theta$	$\cos(180^\circ+\theta)\equiv-\cos\theta$	$\tan(180^\circ+\theta)\equiv\tan\theta$
$\sin(270^\circ-\theta)\equiv-\cos\theta$	$\cos(270^\circ-\theta)\equiv-\sin\theta$	$\tan(270^\circ-\theta)\equiv 1/\tan\theta$
$\sin(270^\circ+\theta)\equiv-\cos\theta$	$\cos(270^\circ+\theta)\equiv\sin\theta$	$\tan(270^\circ+\theta)\equiv -1/\tan\theta$
$\sin(360^\circ-\theta)\equiv-\sin\theta$	$\cos(360^\circ-\theta)\equiv\cos\theta$	$\tan(360^\circ-\theta)\equiv-\tan\theta$
$\sin(360^\circ+\theta)\equiv\sin\theta$	$\cos(360^\circ+\theta)\equiv\cos\theta$	$\tan(360^\circ+\theta)\equiv\tan\theta$

Trigonometry: Negative Angle Formulae

$\sin(-\theta)=-\sin\theta$ $\quad\cos(-\theta)=\cos\theta$ $\quad\tan(-\theta)=-\tan\theta$

Trigonometry: Double Angle Formulae

MF26

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1-\tan^2 A}$$

Trigonometry: Half Angle Formulae

$\sin^2\frac{A}{2}\equiv\frac{1}{2}(1-\cos A)$ $\quad\cos^2\frac{A}{2}\equiv\frac{1}{2}(1+\cos A)$ $\quad\tan^2\frac{A}{2}\equiv\frac{1-\cos A}{1+\cos A}$

Trigonometry: t Formulae

$\sin A\equiv\frac{2t}{1+t^2}$ $\quad\cos A\equiv\frac{1-t^2}{1+t^2}$ $\quad\tan A\equiv\frac{2t}{1-t^2}$ $\quad$ where $t\equiv\tan\left(\frac{A}{2}\right)$

Trigonometry: Triple Angle Formulae

$\sin 3A\equiv 3\sin A-4\sin^3 A$ $\quad\cos 3A\equiv 4\cos^3 A-3\cos A$ $\quad\tan 3A\equiv\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$

Trigonometry: Sum-to-Product Formulae

MF26

$$\sin P+\sin Q\equiv 2\sin\tfrac{1}{2}(P+Q)\cos\tfrac{1}{2}(P-Q)$$

$$\sin P-\sin Q\equiv 2\cos\tfrac{1}{2}(P+Q)\sin\tfrac{1}{2}(P-Q)$$

$$\cos P+\cos Q\equiv 2\cos\tfrac{1}{2}(P+Q)\cos\tfrac{1}{2}(P-Q)$$

$$\cos P-\cos Q\equiv -2\sin\tfrac{1}{2}(P+Q)\sin\tfrac{1}{2}(P-Q)$$

Trigonometry: Product-to-Sum Formulae

$$2\sin A\cos B\equiv\sin(A+B)+\sin(A-B)$$

$$2\cos A\sin B\equiv\sin(A+B)-\sin(A-B)$$

$$2\cos A\cos B\equiv\cos(A+B)+\cos(A-B)$$

$$-2\sin A\sin B\equiv\cos(A+B)-\cos(A-B)$$

Trigonometry: General Solution

$\sin\theta=k\Rightarrow\theta=180°n+(-1)^n\alpha$ where α is the principal value of $\sin^{-1}k$.
$\cos\theta=k\Rightarrow\theta=360°n\pm\alpha$ where α is the principal value of $\cos^{-1}k$.
$\tan\theta=k\Rightarrow\theta=180°n+\alpha$ where α is the principal value of $\tan^{-1}k$.

Differentiation: Product Rule, Quotient Rule & Higher Derivative

Product Rule	Quotient Rule	Higher Derivative
$\frac{\mathrm{d}}{\mathrm{d}x}(uv)=u\frac{\mathrm{d}v}{\mathrm{d}x}+v\frac{\mathrm{d}u}{\mathrm{d}x}$	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u}{v}\right)=\frac{v\frac{\mathrm{d}u}{\mathrm{d}x}-u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$	$\frac{\mathrm{d}^{n+1}y}{\mathrm{d}x^{n+1}}=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}^n y}{\mathrm{d}x^n}\right)$

Differentiation: Reciprocal Rule, Chain Rule & Parametric Differentiation

Reciprocal Rule	Chain Rule	Parametric Differentiation
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\cdot\frac{\mathrm{d}u}{\mathrm{d}x}$	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$

Differentiation: Algebraic, Exponential, Log & Trigonometric Functions

$\mathrm{f}(x)$	$\mathrm{f}'(x)$	$\mathrm{f}(x)$	$\mathrm{f}'(x)$
a	0	x^n	nx^{n-1}
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
a^x	$a^x \ln a$	e^x	e^x
$\log_b x$	$\frac{1}{x\ln b}$	$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$	$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cos x$	$-\sin x$	$\sec x$	$\sec x \tan x$
$\tan x$	$\sec^2 x$	$\cot x$	$-\operatorname{cosec}^2 x$

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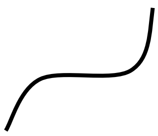
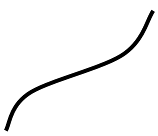
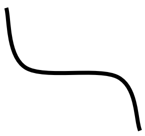
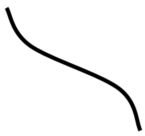
Differentiation: Inverse Trigonometric Functions

MF26

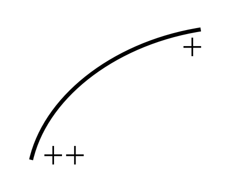
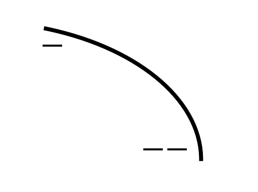
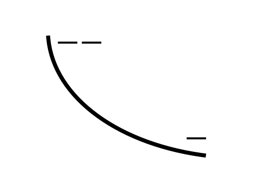
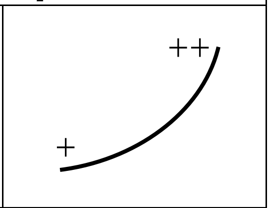
$\mathrm{f}(x)$	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
$\mathrm{f}'(x)$	$\frac{1}{\sqrt{1-x^2}}$	$-\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$

Differentiation: (Strictly) Increasing / Decreasing

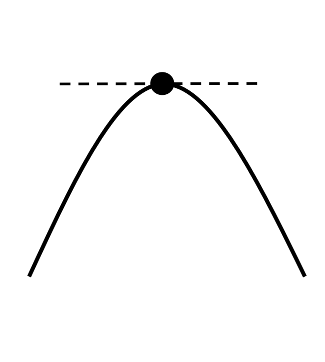
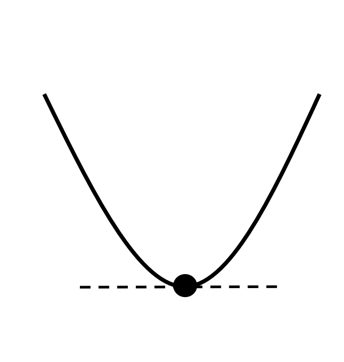
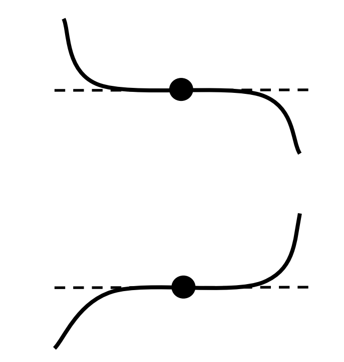
Given: $x_1 < x_2$,

	Increasing	Strictly Increasing	Decreasing	Strictly Decreasing
$\mathrm{f}(x)$				
$\mathrm{f}(x)$	$\mathrm{f}(x_1)\le \mathrm{f}(x_2)$	$\mathrm{f}(x_1)< \mathrm{f}(x_2)$	$\mathrm{f}(x_1)\ge \mathrm{f}(x_2)$	$\mathrm{f}(x_1)> \mathrm{f}(x_2)$
$\mathrm{f}'(x)$	≥ 0	>0	≤ 0	<0

Differentiation: Curvature

	Concave Downward		Concave Upward	
$f(x)$				
$f'(x)$	+ ↓	– ↓	– ↑	+ ↑
$f''(x)$	–		+	

Differentiation: Stationary Points

	Maximum Turning Point			Minimum Turning Point			Stationary Point of Inflexion		
	• The highest point in the neighbouring region.			• The lowest point in the neighbouring region.			• The point with the lowest / highest gradient in the neighbouring region. The word "Stationary" suggests that the gradient at that point is zero.		
$f(x)$									
$f'(x)$	a^-	a	a^+	a^-	a	a^+	a^-	a	a^+
	+	0	–	–	0	+	– / +	0	– / +
	Change in Sign						No change in Sign		
$f''(x)$		–			+		+ / –	0	– / +
							Change in Sign		

Differentiation: Rate of Change / Maxima & Minima involving Volume & Total Surface Area

	Cylinder	Cone	Sphere
Volume	$\pi r^2 h$	$\frac{1}{3}\pi r^2 h$	$\frac{4}{3}\pi r^3$
Total Surface Area	$2\pi r(r+h)$	$\pi r(r+l)$	$4\pi r^2$

Integration: Standard Forms

$\int f'(x)[f(x)]^n \, dx = \dfrac{[f(x)]^{n+1}}{n+1} + c \quad (n \neq -1)$	$\int (ax+b)^n \, dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c \quad (n \neq -1)$
$\int \dfrac{f'(x)}{f(x)} \, dx = \ln\lvert f(x)\rvert + c$	$\int \dfrac{1}{ax+b} \, dx = \dfrac{1}{a}\ln\lvert ax+b\rvert + c$
$\int \dfrac{f'(x)}{\sqrt{f(x)}} \, dx = 2\sqrt{f(x)} + c$	$\int \dfrac{1}{\sqrt{ax+b}} \, dx = \dfrac{2}{a}\sqrt{ax+b} + c$
$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$	$\int e^{ax+b} dx = \dfrac{1}{a} e^{ax+b} + c$

Integration: Special Fractions

MF26

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$	$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{a+x}{a-x}\right) \quad (\lvert x\rvert < a)$	$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{x-a}{x+a}\right) \quad (x > a)$

* Arbitrary constants c are omitted. ; a denotes a positive constant.

Integration: Trigonometric Functions

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\cos x$	$\sin x$	$\sec x \tan x$	$\sec x$
$\sin x$	$-\cos x$	$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\sec^2 x$	$\tan x$	$\operatorname{cosec}^2 x$	$-\cot x$

* Arbitrary constants c are omitted.

MF26

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\tan x$	$\ln(\sec x) \quad \left(\lvert x\rvert < \frac{1}{2}\pi\right)$	$\cot x$	$\ln(\sin x) \quad (0 < x < \pi)$
$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x + \cot x) \quad (0 < x < \pi)$	$\sec x$	$\ln(\sec x + \tan x) \quad \left(\lvert x\rvert < \frac{1}{2}\pi\right)$

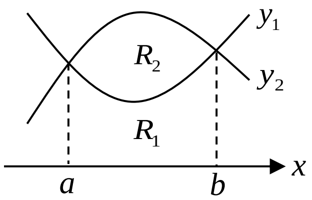
* Arbitrary constants c are omitted. ; a denotes a positive constant.

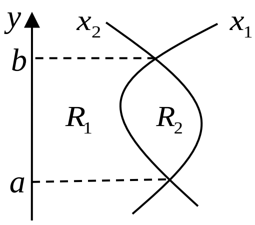
Integration: By Part

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

Log **I**nv Trigo **A**lgebra **T**rigo **E**xp

Integration: Area and Volume

	Area (along the x-axis)	Volume of Revolution (about the x-axis)
	$A_{R_1} = \int_a^b y_1 \, dx$	$V_{R_1} = \pi \int_a^b {y_1}^2 \, dx$
	$A_{R_2} = \int_a^b (y_2 - y_1) \, dx$	$V_{R_2} = \pi \int_a^b ({y_2}^2 - {y_1}^2) \, dx$

	Area (along the y-axis)	Volume of Revolution (about the y-axis)
	$A_{R_1} = \int_a^b x_1 \, dy$	$V_{R_1} = \pi \int_a^b {x_1}^2 \, dy$
	$A_{R_2} = \int_a^b (x_2 - x_1) \, dy$	$V_{R_2} = \pi \int_a^b ({x_2}^2 - {x_1}^2) \, dy$

Integration: Finding the Area using Parametric Equations

Given: $x = f(t)$ and $y = g(t)$,

Area (along the x-axis)	$\int_a^b y \, dx = \int_{t_1}^{t_2} g(t) \frac{dx}{dt} dt$ where $a = f(t_1)$ and $b = f(t_2)$
Area (along the y-axis)	$\int_a^b x \, dy = \int_{t_1}^{t_2} f(t) \frac{dy}{dt} dt$ where $a = g(t_1)$ and $b = g(t_2)$

Differential Equation: Direct Integration vs Separable Variables

Direct Integration (Once)	$\frac{dy}{dx} = f(x) \Rightarrow y = \int f(x) \, dx$
Direct Integration (Twice)	$\frac{d^2y}{dx^2} = f(x) \Rightarrow y = \int \left(\int f(x) \, dx \right) dx$
Separable Variables	$\frac{dy}{dx} = f(y) \Rightarrow \int \frac{1}{f(y)} dy = \int dx$

Differential Equation: Modelling Real Life Problem Situation

$$\text{Net Rate of Change} = \text{Rate of Increase} - \text{Rate of Decrease}$$

Example: Liquid flows into a cylindrical container at a constant rate, and flows out at a rate which is proportional to the depth of water (x) in the container.

Volume of Liquid, $V = \pi r^2 x$ Rate of Increase $= k_1$ Rate of Decrease $= k_2 x$ $(k_1, k_2 > 0)$

$$\therefore \frac{dV}{dt} = \pi r^2 \frac{dx}{dt} = k_1 - k_2 x$$

Vectors: Dot Product vs Cross Product

Dot Product	Cross Product
$\mathbf{a}\cdot\mathbf{b}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\cdot\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=a_1b_1+a_2b_2+a_3b_3$	MF26 $\mathbf{a}\times\mathbf{b}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$
$\mathbf{a}\cdot\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$	$\mathbf{a}\times\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\,\hat{\mathbf{n}}$
$\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$	$\mathbf{a}\times\mathbf{b}=-\mathbf{b}\times\mathbf{a}$
$\mathbf{a}\cdot\mathbf{a}=\lvert\mathbf{a}\rvert^2$	$\mathbf{a}\times\mathbf{a}=\begin{pmatrix}0\\0\\0\end{pmatrix}\Rightarrow\lvert\mathbf{a}\times\mathbf{a}\rvert=0$

Vectors: Unit Vector

$$\hat{\mathbf{a}}=\frac{\mathbf{a}}{\lvert\mathbf{a}\rvert}=\frac{a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}}{\sqrt{a_1^2+a_2^2+a_3^2}}$$

Vectors: Parallel vs Perpendicular Vectors

Parallel	Perpendicular
$\mathbf{a}=\lambda\mathbf{b}$	$\mathbf{a}\cdot\mathbf{b}=0$

Vectors: Ratio Theorem & Mid-Point Theorem

MF26 The point dividing AB in the ratio $\lambda:\mu$ has position vector $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$.

The point dividing AB in the ratio $1:1$ has position vector $\dfrac{\mathbf{a}+\mathbf{b}}{2}$.

Vectors: Equation of a Straight Line

If A is a point with position vector $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has

Cartesian Equation: $\dfrac{x-a_1}{b_1}=\dfrac{y-a_2}{b_2}=\dfrac{z-a_3}{b_3}(=\lambda)$

Vector Equation: $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$ where $\lambda\in\Re$

Vectors: Equation of a Plane

The plane through A with normal vector $\mathbf{n}=n_1\mathbf{i}+n_2\mathbf{j}+n_3\mathbf{k}$ has

Cartesian Equation: $n_1x+n_2y+n_3z+d=0$ where $d=-\mathbf{a}\cdot\mathbf{n}$

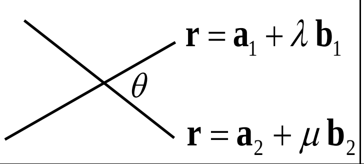
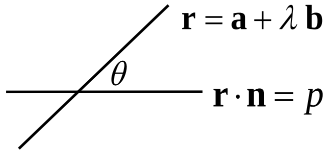
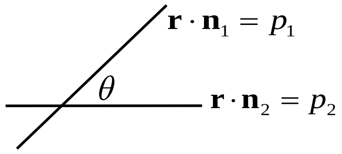
Parametric Equation: $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$ where $\mathbf{b}$ and $\mathbf{c}$ are non-parallel vectors which are both parallel to the plane.

Scalar Product Equation: $\mathbf{r}\cdot\mathbf{n}=p$ where $p=\mathbf{a}\cdot\mathbf{n}$

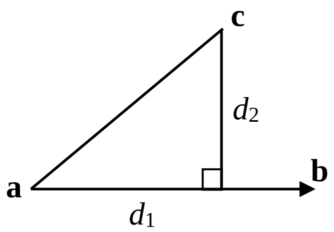
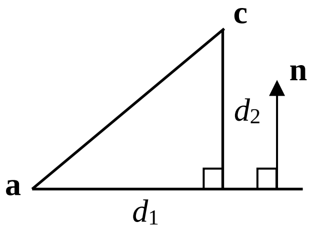
Vectors: Area

Triangle	Parallelogram
$\frac{1}{2}\lvert \mathbf{a}\times\mathbf{b}\rvert$	$\lvert \mathbf{a}\times\mathbf{b}\rvert$

Vectors: Acute Angle

2 Lines	1 Line 1 Plane	2 Planes
		
$\cos\theta=\frac{\lvert\mathbf{b}_1\cdot\mathbf{b}_2\rvert}{\lvert\mathbf{b}_1\rvert\lvert\mathbf{b}_2\rvert}$	$\sin\theta=\frac{\lvert\mathbf{b}\cdot\mathbf{n}\rvert}{\lvert\mathbf{b}\rvert\lvert\mathbf{n}\rvert}$	$\cos\theta=\frac{\lvert\mathbf{n}_1\cdot\mathbf{n}_2\rvert}{\lvert\mathbf{n}_1\rvert\lvert\mathbf{n}_2\rvert}$

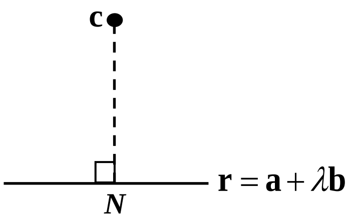
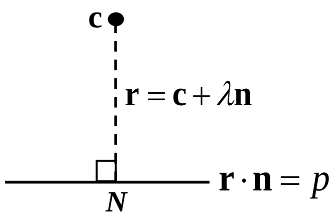
Vectors: Length of Projection vs Perpendicular Distance

	Line	Plane
		
Length of Projection: d_1	$\lvert(\mathbf{c}-\mathbf{a})\cdot\hat{\mathbf{b}}\rvert$	$\lvert(\mathbf{c}-\mathbf{a})\times\hat{\mathbf{n}}\rvert$
Perpendicular Distance: d_2	$\lvert(\mathbf{c}-\mathbf{a})\times\hat{\mathbf{b}}\rvert$	$\lvert(\mathbf{c}-\mathbf{a})\cdot\hat{\mathbf{n}}\rvert$

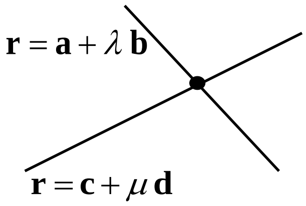
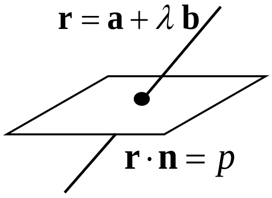
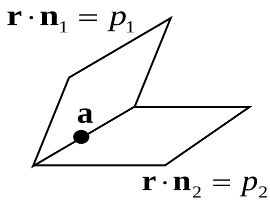
Vectors: Perpendicular Distance from the Origin

Line	Plane
$\lvert\mathbf{a}\times\hat{\mathbf{b}}\rvert$	$\frac{\lvert p\rvert}{\lvert\mathbf{n}\rvert}$

Vectors: Foot of Perpendicular

Line	Plane
	
$\mathbf{a}+\lambda_1\mathbf{b}$ where $(\mathbf{a}+\lambda_1\mathbf{b}-\mathbf{c})\cdot\mathbf{b}=0$	$\mathbf{c}+\lambda_1\mathbf{n}$ where $(\mathbf{c}+\lambda_1\mathbf{n})\cdot\mathbf{n}=p$

Vectors: Point / Line of Intersection

2 Lines	1 Line 1 Plane	2 Planes
		
$\mathbf{a} + \lambda_1\mathbf{b}$ or $\mathbf{c} + \mu_1\mathbf{d}$ where $\mathbf{a} + \lambda_1\mathbf{b} = \mathbf{c} + \mu_1\mathbf{d}$	$\mathbf{a} + \lambda_1\mathbf{b}$ where $(\mathbf{a} + \lambda_1\mathbf{b}) \cdot \mathbf{n} = p$	$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{n}_1 \times \mathbf{n}_2)$ where $\mathbf{a} \cdot \mathbf{n}_1 = p_1$ and $\mathbf{a} \cdot \mathbf{n}_2 = p_2$

Complex Numbers: Cartesian , Polar & Exponential Forms

Cartesian Form	Polar Form	Exponential Form
$z = x + iy$	$z = r(\cos\theta + i\sin\theta)$	$z = re^{i\theta}$

Note: $r = \|z\| \geq 0$ and $-\pi < \theta = \arg(z) \leq \pi$

Complex Numbers: Modulus & Argument

$z = x + iy$	x	y	$\|z\|$	$\arg z$
1st quadrant	+	+	$\sqrt{x^2 + y^2}$	$\tan^{-1}\left(\frac{y}{x}\right)$
2nd quadrant	−	+		$\pi - \tan^{-1}\left\|\frac{y}{x}\right\|$
3rd quadrant	−	−		$\tan^{-1}\left\|\frac{y}{x}\right\| - \pi$
4th quadrant	+	−		$-\tan^{-1}\left\|\frac{y}{x}\right\|$

Note: $-\pi < \arg z \leq \pi$

w	$\|w\|$	$\arg w$
$z_1 z_2$	$\|z_1\|\|z_2\|$	$\arg z_1 + \arg z_2$
$\frac{z_1}{z_2}$	$\frac{\|z_1\|}{\|z_2\|}$	$\arg z_1 - \arg z_2$
z^n	$\|z\|^n$	$n \arg z$

Note: $-\pi < \arg w \leq \pi$

Complex Numbers: Conjugate

$z = x + iy \Rightarrow z^* = x - iy$		$\|z^*\| = \|z\|$	$\arg(z^*) = -\arg(z)$
$z + z^* = 2\,\mathrm{Re}(z)$	$z - z^* = 2i\,\mathrm{Im}(z)$	$(z^*)^* = z$	$(z^n)^* = (z^*)^n$
$(zw)^* = z^* w^*$	$\left(\frac{z}{w}\right)^* = \frac{z^*}{w^*}$	$zz^* = \|z\|^2$	$\frac{z}{z^*} = \cos 2\theta + i\sin 2\theta$

Complex Numbers: Polynomial Equation with Real Coefficients

Given $x+iy$ is a root of a polynomial equation with real coefficients, then its conjugate $x-iy$ is also one of the roots. These two roots form a quadratic factor as given below.

$$[z-(x+iy)][z-(x-iy)]=z^2-2xz+x^2+y^2$$

Complex Numbers: Special Results involving Exponential Form

$e^{i\theta}+e^{-i\theta}=2\cos\theta$	$e^{i\theta}-e^{-i\theta}=2i\sin\theta$
$e^{i\theta}+1=\left(2\cos\frac{\theta}{2}\right)e^{i\left(\frac{\theta}{2}\right)}$	$e^{i\theta}-1=\left(2\sin\frac{\theta}{2}\right)e^{i\left(\frac{\theta}{2}+\frac{\pi}{2}\right)}$

P&C: Basic Selection and Arrangement

Number of ways of selecting *r* objects out of *n* different objects without replacement			${}_n\mathrm{C}_r$
Number of ways of arranging *r* different objects in a row			$r!$
Number of ways of selecting *r* objects out of *n* different objects without replacement, followed by arrangement in a row			${}_n\mathrm{P}_r$
${}_n\mathrm{C}_r=\dfrac{n!}{r!(n-r)!}$	$r!=r(r-1)(r-2)\cdots3\cdot2\cdot1$	${}_n\mathrm{P}_r=\dfrac{n!}{(n-r)!}$	${}_n\mathrm{P}_r={}_n\mathrm{C}_r\times r!$

P&C: Arrangement with Repeated Objects

Number of ways of arranging *r* objects in a row, with k_1 identical objects and another k_2 identical objects … etc.	$\dfrac{r!}{k_1!k_2!\cdots}$

P&C: Arrangement where Repetition is allowed

Number of ways of forming a row with *r* objects, if there are *n* different objects available and repetition is allowed	n^r

P&C: Circular Permutation

Number of ways of arranging *n* different objects in a circular manner	Seats are identical	$(n-1)!$
	Seats are different / numbered	$(n-1)!\times n$

Probability: Standard Results

$0\le \mathrm{P}(A)=\dfrac{\mathrm{n}(A)}{\mathrm{n}(S)}\le 1$
$\mathrm{P}(A')=1-\mathrm{P}(A)$
$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$
$\mathrm{P}(A\cup B)=1-\mathrm{P}(A'\cap B')$
$\mathrm{P}(A\cap B)=1-\mathrm{P}(A'\cup B')$
$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(A'\cap B)=\mathrm{P}(B)+\mathrm{P}(A\cap B')$
$\mathrm{P}(A\cap B)=\mathrm{P}(A)-\mathrm{P}(A\cap B')=\mathrm{P}(B)-\mathrm{P}(A'\cap B)$

Probability: Conditional Probability

$$P(A\mid B)=\frac{P(A\cap B)}{P(B)}=\frac{n(A\cap B)}{n(B)}$$

Probability: Mutually Exclusive Events

$P(A\cap B)=0$	$P(A\cup B)=P(A)+P(B)$

Probability: Independent Events

$P(A\cap B)=P(A)P(B)$	$P(A\mid B)=P(A)$	$P(B\mid A)=P(B)$

Random Variable: Expectation (Mean), Variance & Standard Deviation

$\mu=E(X)=\dfrac{\sum(x\cdot f(x))}{\sum f(x)}=\sum(x\cdot P(X=x))$	$\sigma^2=\mathrm{Var}(X)=\dfrac{\sum((x-\mu)^2\cdot f(x))}{\sum f(x)}=\sum((x-\mu)^2\cdot P(X=x))$
$\mathrm{Var}(X)=E(X^2)-(E(X))^2$	$\sigma=\sqrt{\mathrm{Var}(X)}$
$E(a)=a$	$\mathrm{Var}(a)=0$
$E(aX\pm b)=aE(X)\pm b$	$\mathrm{Var}(aX\pm b)=a^2\mathrm{Var}(X)$
$E(aX\pm bY)=aE(X)\pm bE(Y)$	$\mathrm{Var}(aX\pm bY)=a^2\mathrm{Var}(X)+b^2\mathrm{Var}(Y)$

Binomial Distribution: Probability, Mean and Variance

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Distribution of X	$P(X=x)$	Mean	Variance
Binomial $B(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$

Binomial Distribution: Probability when x=0

Binomial $B(n,p)$	$P(X=0)=(1-p)^n$

Binomial Distribution: Cumulative Probability

Binomial $B(n,p)$	$P(X\le k)=(1-p)^n+\binom{n}{1}p(1-p)^{n-1}+\binom{n}{2}p^2(1-p)^{n-2}+\ldots\binom{n}{k}p^k(1-p)^{n-k}$

Binomial Distribution: Common Phrases

X is	Original Probability	For binomcdf [GC]
not more than / at most k	$P(X\le k)$	$P(X\le k)$
fewer than k	$P(X<k)$	$P(X\le k-1)$
more than k	$P(X>k)$	$1-P(X\le k)$
at least / not fewer than k	$P(X\ge k)$	$1-P(X\le k-1)$

Note: k is a positive integer.

Normal Distribution: Standardization

$$X \sim \mathrm{N}\left(\mu\,,\sigma^2\right) \Rightarrow Z = \frac{X-\mu}{\sigma} \sim \mathrm{N}(0\,,1)$$

Normal Distribution: Symmetry Properties

Given a is positive and $Z \sim \mathrm{N}(0\,,1)$,

$P(Z > a) = P(Z < -a)$	$P(Z > -a) = P(Z < a)$
$P(\lvert Z\rvert > a) = 2P(Z < -a)$	$P(\lvert Z\rvert < a) = 1 - 2P(Z < -a)$

Note: The above symmetry properties are also applicable to other Normal Distributions with mean $\mu = 0$.

Normal Distribution: Linear Combinations of Random Variables

X	$\mathrm{N}\left(\mu\,,\sigma^2\right)$
mX	$\mathrm{N}\left(m\mu\,,m^2\sigma^2\right)$
$X_1 + X_2 + \ldots + X_n$	$\mathrm{N}\left(n\mu\,,n\sigma^2\right)$
$m\left(X_1 + X_2 + \ldots + X_n\right)$	$\mathrm{N}\left(mn\mu\,,m^2n\sigma^2\right)$
$\overline{X} = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$	$\mathrm{N}\left(\mu,\dfrac{\sigma^2}{n}\right)$

Central Limit Theorem (CLT)

Original Distribution	Condition	Approximate Distribution
Non-Normal with mean μ and variance σ^2	$n > 30$	Sample Mean $\overline{X} \sim N\left(\mu,\dfrac{\sigma^2}{n}\right)$
		Sample Sum $X_1 + X_2 + \ldots + X_n \sim N\left(n\mu,n\sigma^2\right)$

Unbiased Estimates:

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$$s^2 = \frac{n}{n-1}\sigma_n^2 = \frac{n}{n-1}\,\frac{\sum\left(x-\bar{x}\right)^2}{n} \Rightarrow s^2 = \frac{1}{n-1}\sum\left(x-\bar{x}\right)^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$$

Given Sample Data		Unbiased Estimate		Remarks
		Mean $\hat{\mu}$	**Variance** $\hat{\sigma}^2 = s^2$	
$\sum(x-a)$	$\sum(x-a)^2$	$\dfrac{\sum(x-a)}{n} + a$	$\dfrac{1}{n-1}\left(\sum(x-a)^2 - \dfrac{\left(\sum(x-a)\right)^2}{n}\right)$	
$\sum x$	$\sum(x-a)^2$	$\dfrac{\sum x}{n}$		$\sum(x-a) = \sum x - na$
$\sum(x-a)$	$\sum x^2$	$\dfrac{\sum(x-a)}{n} + a$	$\dfrac{1}{n-1}\left(\sum x^2 - \dfrac{\left(\sum x\right)^2}{n}\right)$	$\sum x = \sum(x-a) + na$

Hypothesis Testing

Population	Sample Size	Population Variance	Test Statistic	Test
Normal	large $n > 30$	σ^2 known	$Z = \dfrac{\overline{X} - \mu_0}{\sqrt{\sigma^2/n}} \sim N(0,1)$	Z
		σ^2 unknown	$Z = \dfrac{\overline{X} - \mu_0}{\sqrt{s^2/n}} \sim N(0,1)$	
	small $n < 30$	σ^2 known	$Z = \dfrac{\overline{X} - \mu_0}{\sqrt{\sigma^2/n}} \sim N(0,1)$	
non-Normal	large $n > 30$	σ^2 known	$Z = \dfrac{\overline{X} - \mu_0}{\sqrt{\sigma^2/n}} \sim N(0,1)$ approx by CLT	
		σ^2 unknown	$Z = \dfrac{\overline{X} - \mu_0}{\sqrt{s^2/n}} \sim N(0,1)$ approx by CLT	

Note: $s^2 = \dfrac{n}{n-1}\sigma_n^2 = \dfrac{\sum (x-\bar{x})^2}{n-1} = \dfrac{1}{n-1}\left[\sum x^2 - \dfrac{\left(\sum x\right)^2}{n}\right]$

Correlation & Regression: Product Moment Correlation Coefficient

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$$r = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}} = \frac{\sum xy - \dfrac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \dfrac{\left(\sum x\right)^2}{n}\right)\left(\sum y^2 - \dfrac{\left(\sum y\right)^2}{n}\right)}} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

where $S_{xx} = \sum x^2 - \dfrac{\left(\sum x\right)^2}{n}$, $S_{yy} = \sum y^2 - \dfrac{\left(\sum y\right)^2}{n}$ and $S_{xy} = \sum xy - \dfrac{\sum x \sum y}{n}$

Correlation & Regression: Regression Lines

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y on x : $y - \bar{y} = b(x - \bar{x})$ where $b = \dfrac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2} = \dfrac{\sum xy - \dfrac{\sum x \sum y}{n}}{\sum x^2 - \dfrac{\left(\sum x\right)^2}{n}} = \dfrac{S_{xy}}{S_{xx}}$

x on y : $x - \bar{x} = d(y - \bar{y})$ where $d = \dfrac{\sum (x-\bar{x})(y-\bar{y})}{\sum (y-\bar{y})^2} = \dfrac{\sum xy - \dfrac{\sum x \sum y}{n}}{\sum y^2 - \dfrac{\left(\sum y\right)^2}{n}} = \dfrac{S_{xy}}{S_{yy}}$

* b and d are called the Regression Coefficient of regression line of y on x and x on y respectively.
* r, b and d are all in the same signs and $r^2 = bd$.