PERFECT SOLUTION EDUCATION GROUP

JC MATHEMATICS (H2)

STUDY TIPS

NEVER CONFUSE AGAIN: THE ULTIMATE SHORTCUT OF FINDING ANGLES FOR LINES & PLANES

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JC_H2Maths_015

Quick Practice

Plane Π_1 contains the points $\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, $2\mathbf{i}-\mathbf{j}-\mathbf{k}$ and $3\mathbf{i}+2\mathbf{j}+\mathbf{k}$.

Plane Π_2 contains the point $4\mathbf{i}+3\mathbf{j}-2\mathbf{k}$ and the line $r=(\mathbf{i}-2\mathbf{j}-3\mathbf{k})+\lambda(2\mathbf{i}-3\mathbf{j}+\mathbf{k})$.

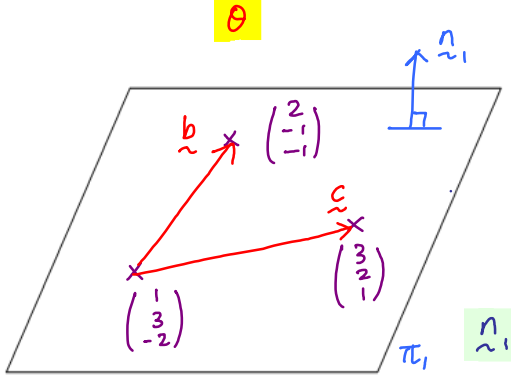
(i) Find the normal vectors of Π_1 and Π_2 respectively.

(ii) Hence, find the acute angle between Π_1 and Π_2.

θ

$\underset{\sim}{n}_1$ $\underset{\sim}{n}_2$

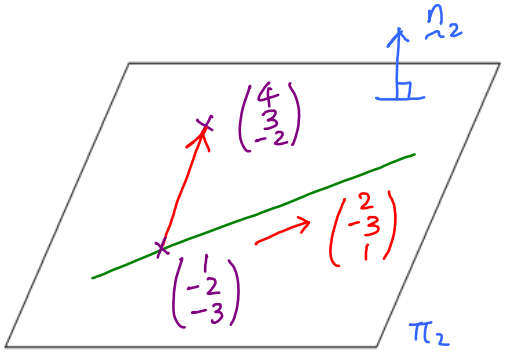
(i)



$\underset{\sim}{b} = \begin{pmatrix}2\\-1\\-1\end{pmatrix} - \begin{pmatrix}1\\3\\-2\end{pmatrix} = \begin{pmatrix}1\\-4\\1\end{pmatrix}$

$\underset{\sim}{c} = \begin{pmatrix}3\\2\\1\end{pmatrix} - \begin{pmatrix}1\\3\\-2\end{pmatrix} = \begin{pmatrix}2\\-1\\3\end{pmatrix}$

$\underset{\sim}{n}_1 = \underset{\sim}{b}\times\underset{\sim}{c} = \begin{pmatrix}1\\-4\\1\end{pmatrix}\times\begin{pmatrix}2\\-1\\3\end{pmatrix} = \begin{pmatrix}-12+1\\-(3-2)\\-1+8\end{pmatrix} = \begin{pmatrix}-11\\-1\\7\end{pmatrix}$ #



$\underset{\sim}{n}_2 = \left[\begin{pmatrix}4\\3\\-2\end{pmatrix} - \begin{pmatrix}1\\-2\\-3\end{pmatrix}\right]\times\begin{pmatrix}2\\-3\\1\end{pmatrix}$

$= \begin{pmatrix}3\\5\\1\end{pmatrix}\times\begin{pmatrix}2\\-3\\1\end{pmatrix}$

$= \begin{pmatrix}5+3\\-(3-2)\\-9-10\end{pmatrix} = \begin{pmatrix}8\\-1\\-19\end{pmatrix}$ #

(ii) $\cos\theta = \dfrac{|\underset{\sim}{n}_1\cdot\underset{\sim}{n}_2|}{|\underset{\sim}{n}_1||\underset{\sim}{n}_2|} = \dfrac{\left|\begin{pmatrix}-11\\-1\\7\end{pmatrix}\cdot\begin{pmatrix}8\\-1\\-19\end{pmatrix}\right|}{\sqrt{121+1+49}\sqrt{64+1+361}} = \dfrac{220}{\sqrt{171}\sqrt{426}} \Rightarrow \theta = 35.4^\circ$ (to 0.1°) *

2 Lines	1 Line 1 Plane	2 Planes
$\mathbf{r}=\mathbf{a}_1+\lambda\mathbf{b}_1$, $\mathbf{r}=\mathbf{a}_2+\mu\mathbf{b}_2$, θ	$\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}$, $\mathbf{r}\cdot\mathbf{n}=p$, θ	$\mathbf{r}\cdot\mathbf{n}_1=p_1$, $\mathbf{r}\cdot\mathbf{n}_2=p_2$, θ
$\cos\theta = \dfrac{\lvert\mathbf{b}_1\cdot\mathbf{b}_2\rvert}{\lvert\mathbf{b}_1\rvert\lvert\mathbf{b}_2\rvert}$	$\sin\theta = \dfrac{\lvert\mathbf{b}\cdot\mathbf{n}\rvert}{\lvert\mathbf{b}\rvert\lvert\mathbf{n}\rvert}$	$\cos\theta = \dfrac{\lvert\mathbf{n}_1\cdot\mathbf{n}_2\rvert}{\lvert\mathbf{n}_1\rvert\lvert\mathbf{n}_2\rvert}$

* ∠ between the same type of objects ⇒ Cosθ, otherwise ⇒ sinθ
(2 lines or 2 planes) (1 line 1 plane)

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Explanation

There is a simple and reliable shortcut for determining which trigonometric function to use when finding angles in three-dimensional Vectors.

The rule is based entirely on whether the geometric objects are of the same type or mixed types. When calculating the angle between two identical objects, such as two lines or two planes, you always employ the cosine function.

Conversely, when finding the angle between two different types of objects, specifically one line and one plane, you switch to the sine function.

Remembering this fundamental distinction eliminates confusion and ensures you select the correct approach every time.

SIGN UP NOW BEFORE IT'S TOO LATE

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MATHS TUITION BY MR JACKIE LEE

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- Former **MOE Scholar**
- **Winner** of International Mathematics Competitions
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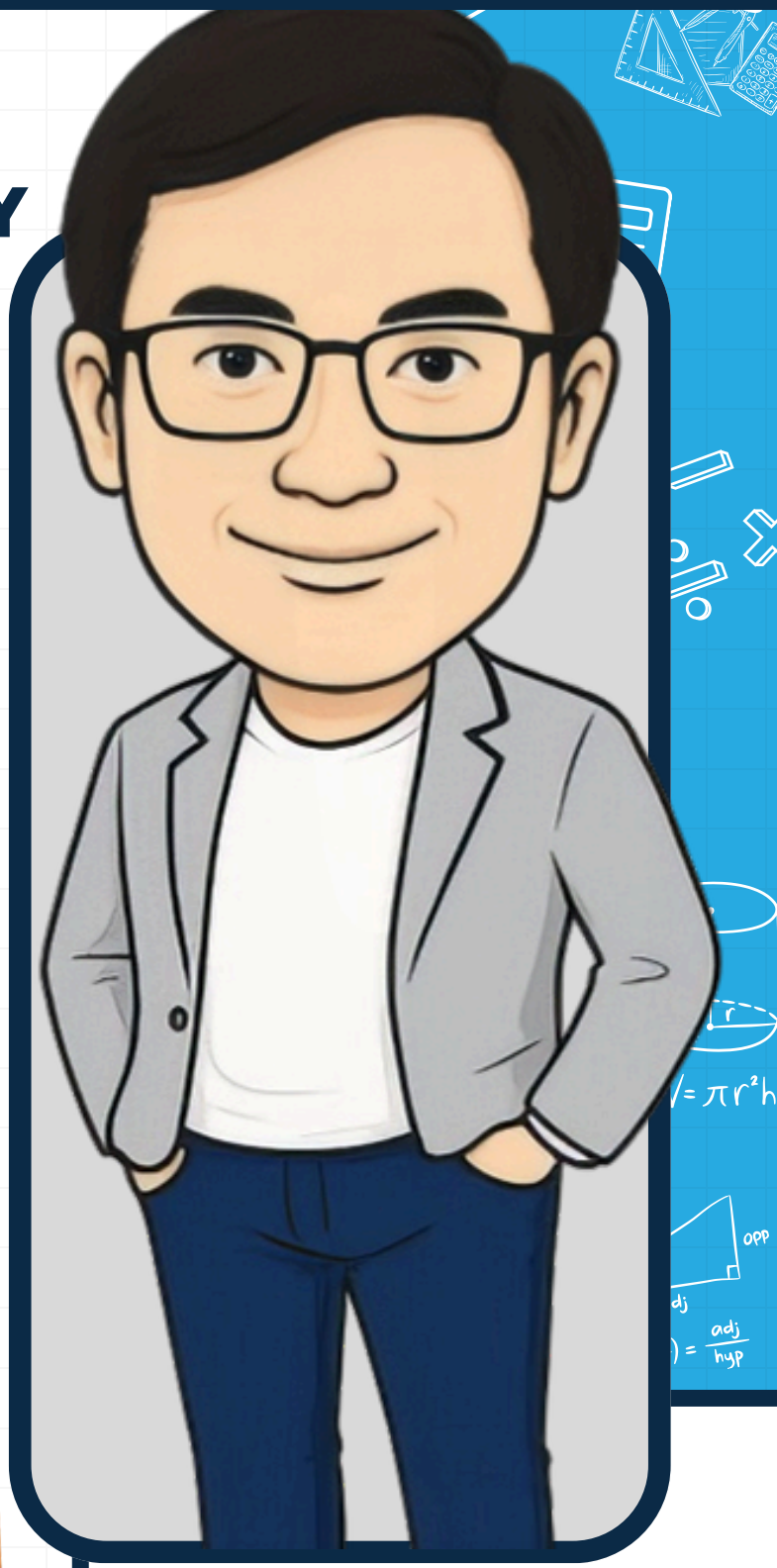

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