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JC MATHEMATICS (H2)

STUDY TIPS

THREE KEY CONSIDERATIONS WHEN FINDING ANGLE BETWEEN TWO VECTORS

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JC_H2Maths_021

5. Scalar Product (or Dot Product)

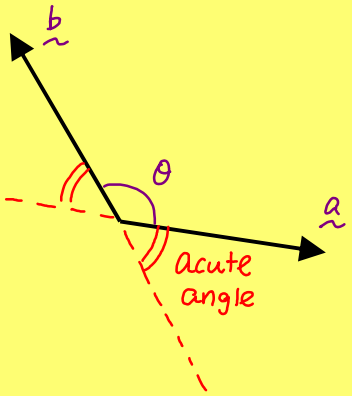
- **Angle between two Vectors**

Given two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$ and the angle between them is θ,

the **Scalar Product** (or **Dot Product**), denoted by $\mathbf{a}\cdot\mathbf{b}$, is defined as follows.

$$\mathbf{a}\cdot\mathbf{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$$

$$\cos\theta = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} \quad \text{use } |\underset{\sim}{a}\cdot\underset{\sim}{b}| \text{ when finding acute angle}, \quad 0^\circ \le \theta \le 180^\circ$$



If $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$, then

$$\mathbf{a}\cdot\mathbf{b} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}\cdot\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = x_1x_2 + y_1y_2 + z_1z_2 .$$

Example 7

Find the acute angle between $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$.

θ

① $\underset{\sim}{a}\cdot\underset{\sim}{b} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = 3 - 6 - 8 = -11$

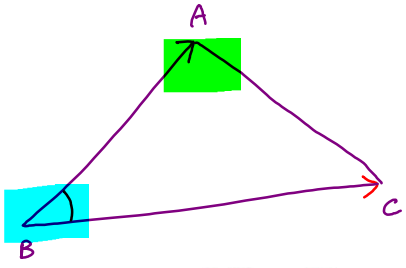
②a $|\underset{\sim}{a}| = \sqrt{3^2+2^2+4^2} = \sqrt{29}$ ②b $|\underset{\sim}{b}| = \sqrt{1^2+3^2+2^2} = \sqrt{14}$

③ $\cos\theta = \dfrac{|\underset{\sim}{a}\cdot\underset{\sim}{b}|}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \dfrac{11}{\sqrt{29}\sqrt{14}} \Rightarrow \theta = 56.9^\circ$ (to 0.1°)

Example 8

It is given that the position vectors A, B and C are $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{c} = 6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ respectively. Find $\angle ABC$.

$\overrightarrow{BA} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 6 \\ -4 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$



① $\overrightarrow{BA}\cdot\overrightarrow{BC} = 15 + 6 - 2 = 19$

② $|\overrightarrow{BA}||\overrightarrow{BC}| = \sqrt{3^2+2^2+1^2}\sqrt{5^2+3^2+2^2} = \sqrt{14}\sqrt{38}$

③ $\cos\angle ABC = \dfrac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|} = \dfrac{19}{\sqrt{14}\sqrt{38}} \Rightarrow \angle ABC = 34.5^\circ$ (to 0.1°)

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Explanation

When calculating the angle between two vectors using their scalar product, three essential rules ensure geometric accuracy.

First, if you are looking specifically for the acute angle, you must force the dot product in the numerator to be positive by removing any negative sign.

Second, if you are determining the actual angle at a vertex, both vectors must start from that exact corner and point outward.

Third, when finding this true angle, the numerator can remain negative if it naturally computes that way, which correctly allows the calculation to yield an obtuse angle.

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MATHS TUITION BY MR JACKIE LEE

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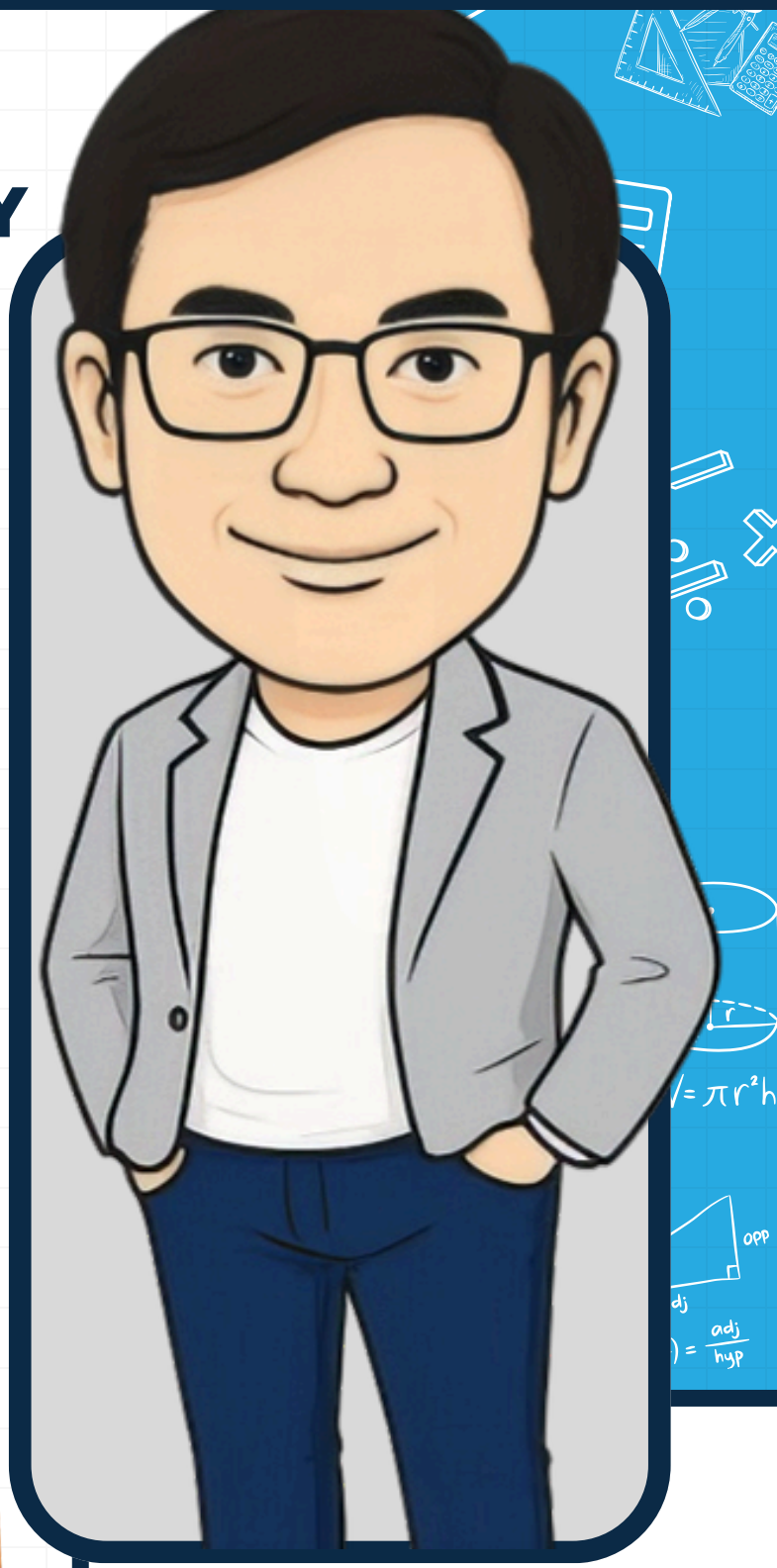

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