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JC MATHEMATICS (H2) STUDY TIPS

**A POWERFUL ALTERNATIVE:
DETERMINING VECTOR
MAGNITUDE VIA SELF-DOT
PRODUCT EXPANSION**

PREPARED BY
Mr Jackie Lee



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JC_H2Maths_041



Example 34

It is given that $|\mathbf{a}| = 2$, $|\mathbf{b}| = 3$ and the angle between them is 60° .

(a) Find the value of $\mathbf{a} \cdot \mathbf{b}$.

$$\underline{a} \cdot \underline{b} = |\underline{a}| |\underline{b}| \cos 60^\circ = (2)(3)\left(\frac{1}{2}\right) = 3$$

(b) Find the value of $(\mathbf{a} + 2\mathbf{b}) \cdot (3\mathbf{a} - \mathbf{b})$.

$$\begin{aligned} & (\underline{a} + 2\underline{b}) \cdot (3\underline{a} - \underline{b}) \\ &= 3 \underline{a} \cdot \underline{a} - \underline{a} \cdot \underline{b} + 6 \underline{b} \cdot \underline{a} - 2 \underline{b} \cdot \underline{b} \\ &= 3 |\underline{a}|^2 + 5 \underline{a} \cdot \underline{b} - 2 |\underline{b}|^2 \\ &= 3(2^2) + 5(3) - 2(3^2) = 9 \end{aligned}$$

(c) Find the values of $|\mathbf{a} + 2\mathbf{b}|$ and $|3\mathbf{a} - \mathbf{b}|$.

$$\begin{aligned} |\underline{a} + 2\underline{b}| &= \sqrt{(\underline{a} + 2\underline{b}) \cdot (\underline{a} + 2\underline{b})} \\ &= \sqrt{\underline{a} \cdot \underline{a} + 2 \underline{a} \cdot \underline{b} + 2 \underline{b} \cdot \underline{a} + 4 \underline{b} \cdot \underline{b}} \\ &= \sqrt{|\underline{a}|^2 + 4 \underline{a} \cdot \underline{b} + 4 |\underline{b}|^2} \\ &= \sqrt{2^2 + 4(3) + 4(3^2)} \\ &= \sqrt{52} = 2\sqrt{13} \end{aligned}$$

$$\begin{aligned} |3\underline{a} - \underline{b}| &= \sqrt{(3\underline{a} - \underline{b}) \cdot (3\underline{a} - \underline{b})} \\ &= \sqrt{9 \underline{a} \cdot \underline{a} - 3 \underline{a} \cdot \underline{b} - 3 \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{b}} \\ &= \sqrt{9 |\underline{a}|^2 - 6 \underline{a} \cdot \underline{b} + |\underline{b}|^2} \\ &= \sqrt{9(2^2) - 6(3) + 3^2} \\ &= \sqrt{27} = 3\sqrt{3} \end{aligned}$$

(d) Find the acute angle between $\mathbf{a} + 2\mathbf{b}$ and $3\mathbf{a} - \mathbf{b}$.

Let θ be the required acute angle.

$$\cos \theta = \frac{|(\underline{a} + 2\underline{b}) \cdot (3\underline{a} - \underline{b})|}{|\underline{a} + 2\underline{b}| |\underline{3a} - \underline{b}|} = \frac{9}{(2\sqrt{13})(3\sqrt{3})}$$

$$\theta = 76.1^\circ \text{ (1 d.p.)}$$



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Explanation

An essential strategy for evaluating vector expressions is to determine the magnitude by calculating the dot product of a vector with itself and subsequently taking the square root of that result.

This method is highly advantageous because it permits the expansion and simplification of complex vector combinations in a manner analogous to standard algebraic manipulation.

Once the expression is reduced to a numerical value, applying the square root at the final stage yields the final magnitude.

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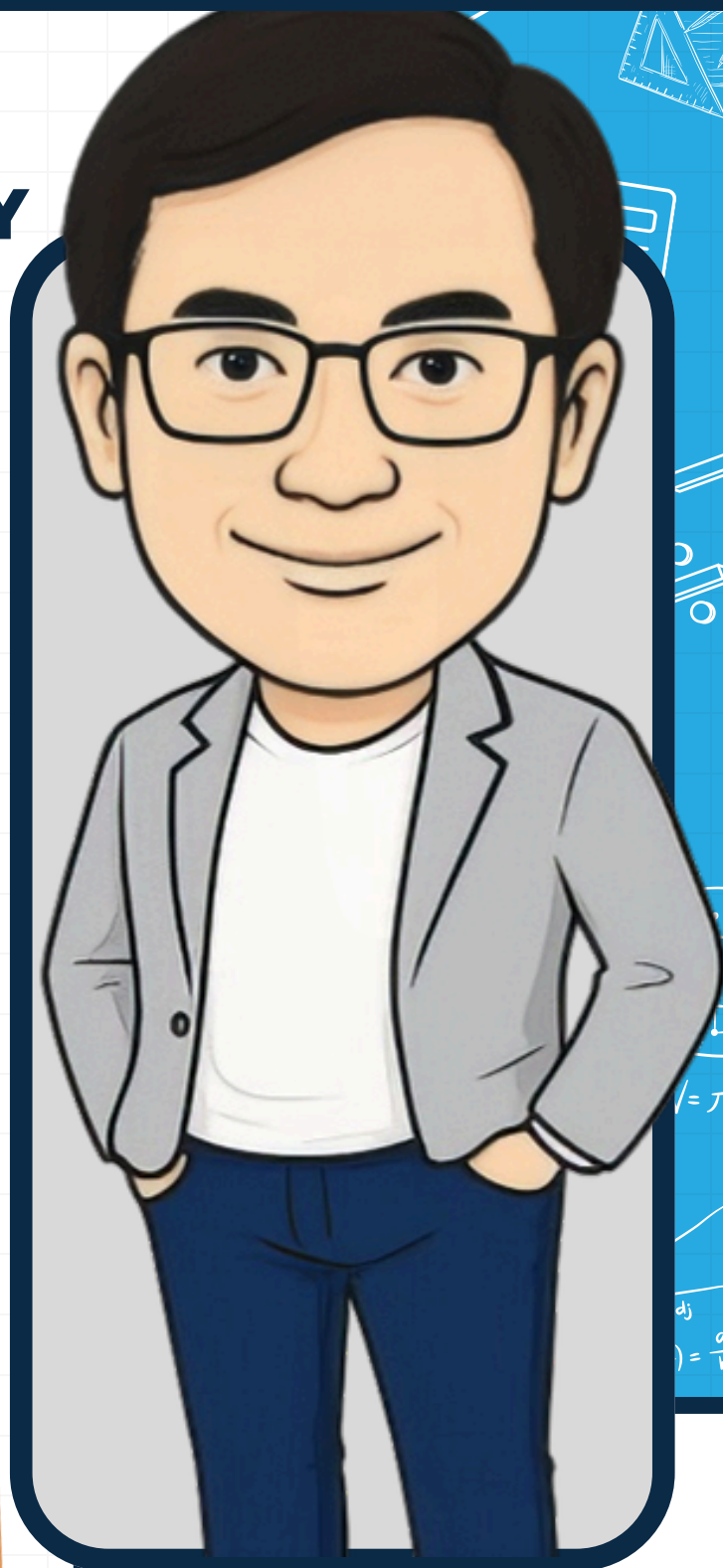
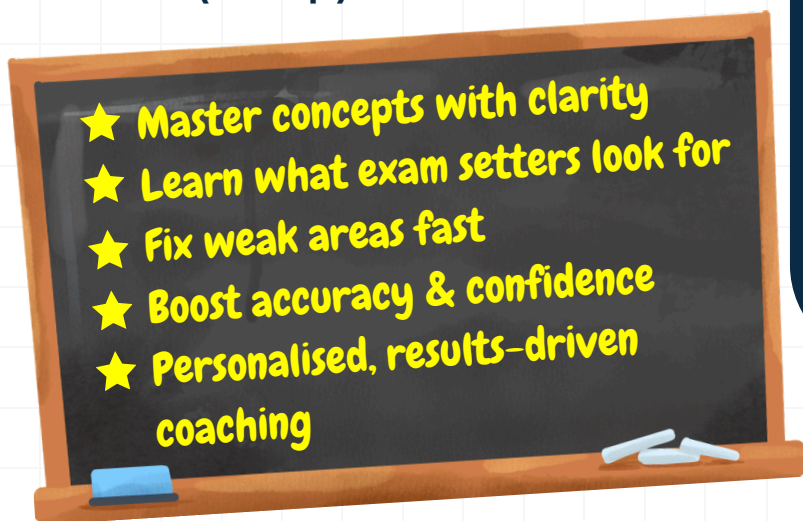
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