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JC MATHEMATICS (H2)

STUDY TIPS

A USEFUL TECHNIQUE IN VECTORS: THE SMART USE OF EQUATING COEFFICIENTS

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It is given that $\mathbf{a}$ and $\mathbf{b}$ are two non-zero and non-parallel vectors.

And if $\alpha\mathbf{a}+\beta\mathbf{b}=\lambda\mathbf{a}+\mu\mathbf{b} \Rightarrow \alpha\mathbf{a}-\lambda\mathbf{a}=\mu\mathbf{b}-\beta\mathbf{b} \Rightarrow (\alpha-\lambda)\mathbf{a}=(\mu-\beta)\mathbf{b}$,

hence, $\alpha-\lambda=0$ and $\mu-\beta=0 \Rightarrow \alpha=\lambda$ and $\beta=\mu$.

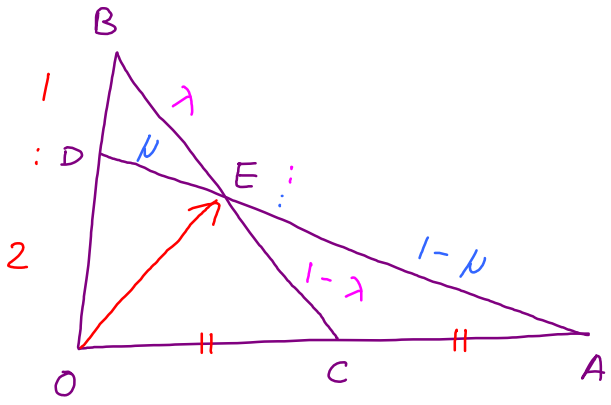
Example 11

Referred to the origin O, the points A and B are such that $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.
The point C lies on OA such that C is the mid-point of O and A. The point D lies on OB such that $OB:OD=3:2$. The lines AD and BC meet at the point E such that $BE=\lambda BC$ and $DE=\mu DA$.

(i) Find $\overrightarrow{OE}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ.

(ii) Find $\overrightarrow{OE}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and μ.

Hence, find the values of λ and μ.



(i) $\overrightarrow{OE} = \dfrac{\lambda\overrightarrow{OC}+(1-\lambda)\overrightarrow{OB}}{\lambda+1-\lambda}$

$= \lambda\left(\frac{1}{2}\underset{\sim}{a}\right)+(1-\lambda)\underset{\sim}{b}$ ★

(ii) $\overrightarrow{OE} = \dfrac{\mu\overrightarrow{OA}+(1-\mu)\overrightarrow{OD}}{\mu+1-\mu}$

$= \mu\underset{\sim}{a}+(1-\mu)\left(\frac{2}{3}\underset{\sim}{b}\right)$ ★

$\therefore \frac{1}{2}\lambda=\mu \Rightarrow \lambda=2\mu$ —— (1)

$1-\lambda=\frac{2}{3}(1-\mu)$ —— (2)

Sub. (1) into (2), $3(1-2\mu)=2(1-\mu)$

$3-6\mu=2-2\mu$

$1=4\mu$

$\mu=1/4$ —— (3) ★

Sub. (3) into (1), $\lambda=2(1/4)=1/2$ ★

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Explanation

The technique of equating coefficients relies on the property of non-parallel guide vectors. When a single position vector is expressed through two different geometric pathways using the same two base vectors, the two expressions must be identical.

Because the base vectors point in entirely different directions, they cannot substitute for or cancel one another. Consequently, the scalar multiplier for the first base vector on one side of the equation must equal its corresponding multiplier on the opposite side.

The same condition applies to the multipliers of the second base vector.

Matching these multipliers independently allows the vector components to be removed, converting a geometric relationship into a system of standard algebraic equations to resolve the unknowns.

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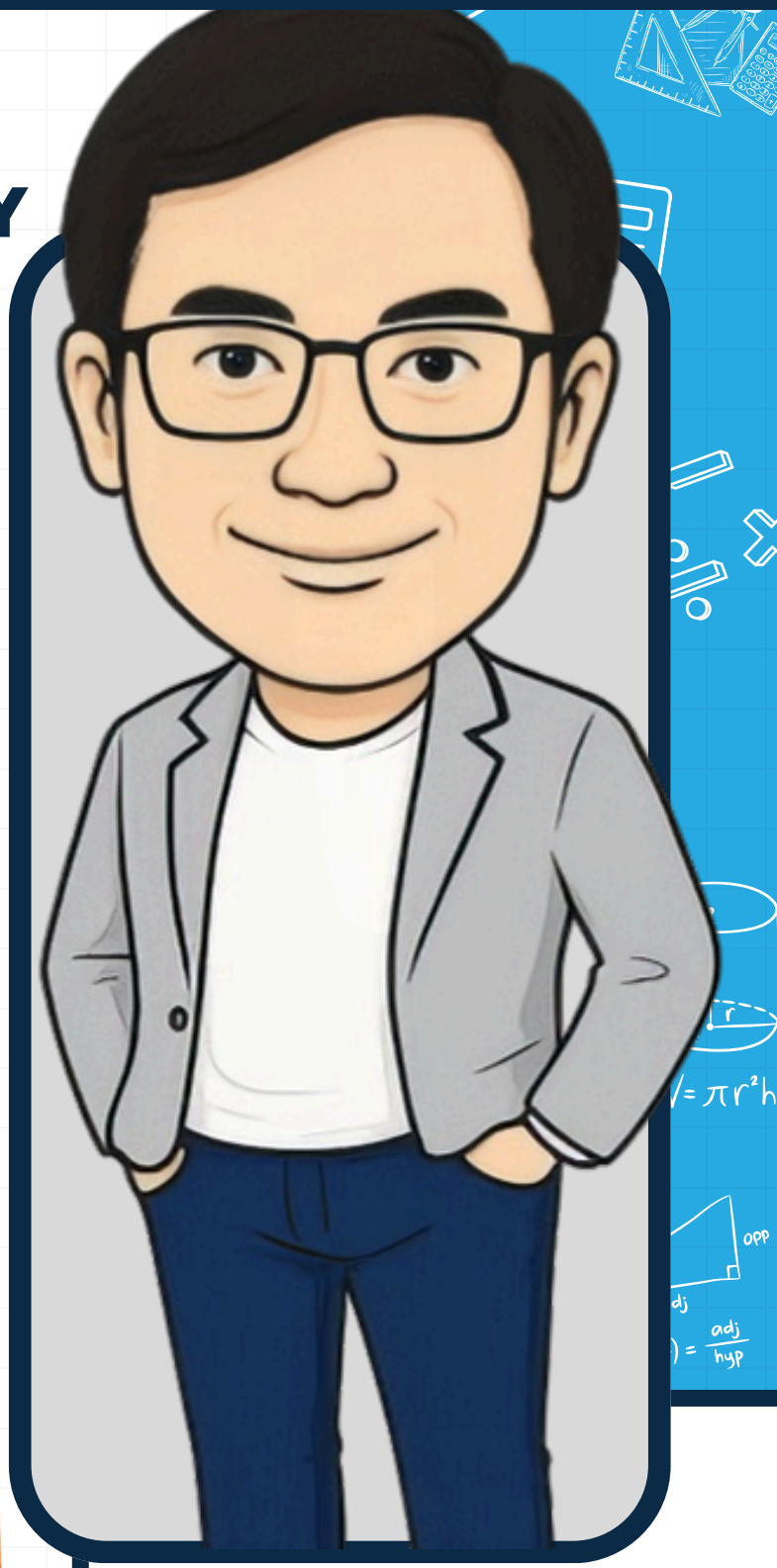

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