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JC MATHEMATICS (H2) STUDY TIPS

**TWO SIMPLE BUT USEFUL
TRICKS IN FORMING THE
VECTOR EQUATION OF A PLANE**

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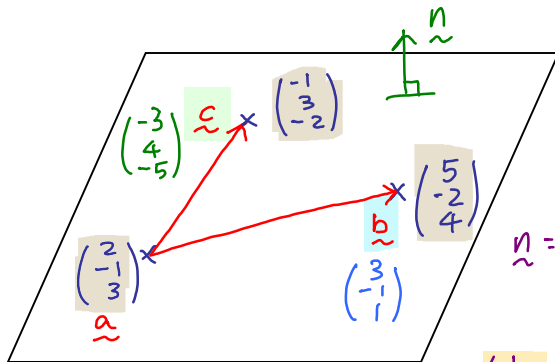
JC_H2Maths_045



• Case 1: 3 Non-Overlapping Points

Example 22

Find a Cartesian equation of the plane defined by $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $5\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$.



$$\underline{b} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \quad \underline{c} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -5 \end{pmatrix}$$

$$\underline{n} = \underline{b} \times \underline{c} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 - 4 \\ -(-15 + 3) \\ 12 - 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 12 \\ 9 \end{pmatrix}$$

Use any one of the given points.

$$\begin{aligned} \underline{r} \cdot \begin{pmatrix} 1 \\ 12 \\ 9 \end{pmatrix} &= \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 12 \\ 9 \end{pmatrix} = 2 - 12 + 27 = 17 \\ &\text{OR} \\ &= \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 12 \\ 9 \end{pmatrix} = 5 - 24 + 36 = 17 \\ &\text{OR} \\ &= \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 12 \\ 9 \end{pmatrix} = -1 + 36 - 18 = 17 \end{aligned} \Rightarrow x + 12y + 9z = 17$$

Quick Practice

Find a Cartesian equation of the plane defined by $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$.

$$\underline{b} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \quad \underline{c} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

$$\underline{n} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 + 2 \\ -(9 - 1) \\ -6 + 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -8 \\ -5 \end{pmatrix} = - \begin{pmatrix} 1 \\ 8 \\ 5 \end{pmatrix}$$

$$\begin{aligned} \underline{r} \cdot \begin{pmatrix} 1 \\ 8 \\ 5 \end{pmatrix} &= \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 8 \\ 5 \end{pmatrix} = 1 + 24 - 10 = 15 \\ &\text{OR} \\ &= \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 8 \\ 5 \end{pmatrix} = 4 + 16 - 5 = 15 \\ &\text{OR} \\ &= \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 8 \\ 5 \end{pmatrix} = 3 - 8 + 20 = 15 \end{aligned} \Rightarrow x + 8y + 5z = 15$$



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Explanation

To find a plane's Cartesian equation, you must find two vectors parallel to the plane and calculate their cross product to obtain the essential perpendicular normal vector.

A key strategy across this process is reducing any of these three vectors by dividing out common factors to keep the numbers as small as possible for easier manipulation.

Once the normal vector is found and used in the scalar product form, you can easily verify your work by substituting another given point into the final equation as a quick accuracy check.

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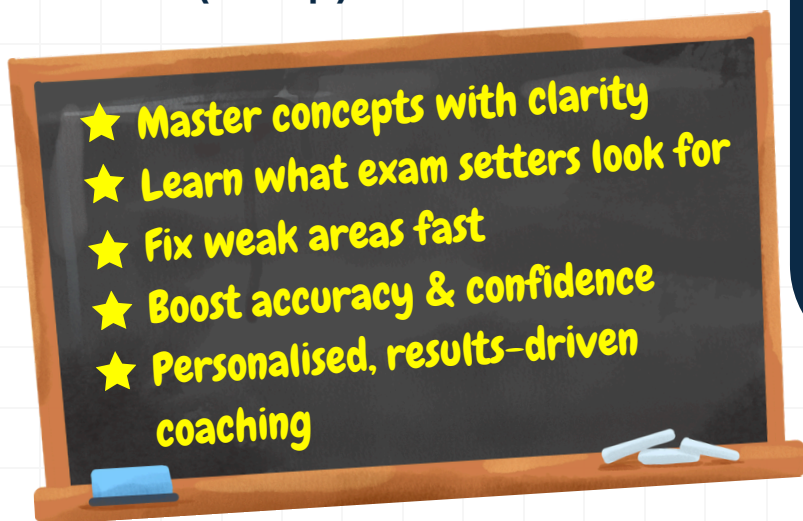
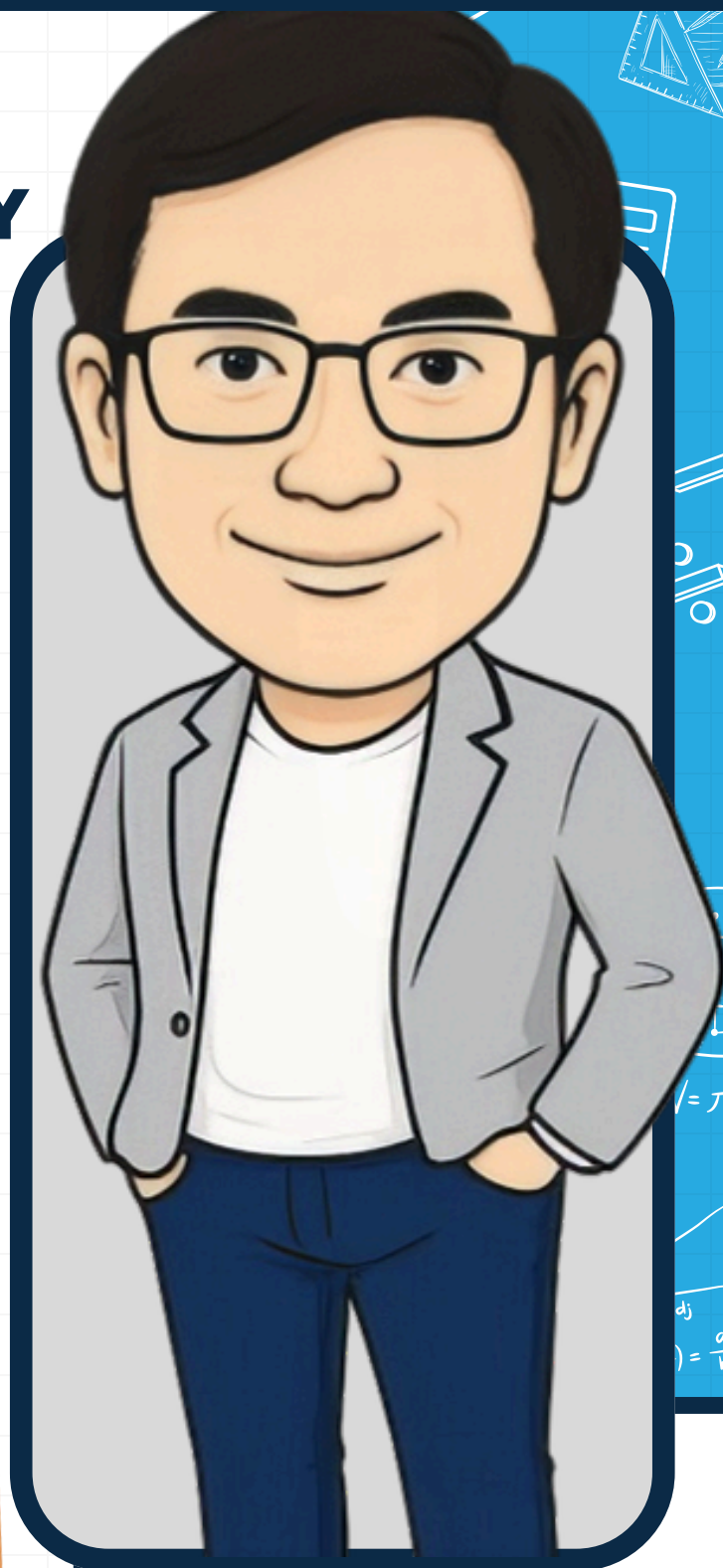
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