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JC MATHEMATICS (H2) STUDY TIPS

**A POWERFUL SHORTCUT:
RESOLVING EXPONENTIALS AND
LOGARITHMS TO SIMPLIFY
INTEGRATION LIMITS**

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JC_H2Maths_047



• Case 2b: Exponential Substitution for Definite Integral

Example 20

Use the substitution $u = e^x$ to find $\int_0^{\ln 2} \frac{e^x}{\sqrt{4 - e^{2x}}} dx$, expressing your answer in exact form.

$$\begin{aligned}
 & x=0, u=e^0=1 \\
 & x=\ln 2, u=e^{\ln 2}=2 \\
 & u=e^x \\
 & \frac{du}{dx}=e^x=u
 \end{aligned}
 \quad \left| \quad
 \begin{aligned}
 & = \int_1^2 \frac{\cancel{u}}{\sqrt{4-u^2}} \left(\frac{dx}{du} \right) du \\
 & \quad \quad \quad \underbrace{\left(\frac{dx}{du} \right)}_{\frac{1}{\frac{du}{dx}} = \frac{1}{u}} du \\
 & = \int_1^2 \frac{1}{\sqrt{2^2-u^2}} du \\
 & = \left[\sin^{-1} \left(\frac{u}{2} \right) \right]_1^2 \\
 & = \sin^{-1}(1) - \sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}
 \end{aligned}$$

Quick Practice

Use the substitution $u = e^x$ to find $\int_0^{\ln 3} \frac{e^x}{e^{2x}+9} dx$, expressing your answer in exact form.

$$\begin{aligned}
 & \text{let } u=e^x. \\
 & x=0, u=e^0=1 \\
 & x=\ln 3, u=e^{\ln 3}=3 \\
 & \frac{du}{dx}=e^x=u \Rightarrow \frac{dx}{du}=\frac{1}{u}
 \end{aligned}
 \quad \left| \quad
 \begin{aligned}
 & \int_0^{\ln 3} \frac{e^x}{e^{2x}+9} dx = \int_1^3 \frac{\cancel{u}}{u^2+9} \left(\frac{dx}{du} \right) du \\
 & \quad \quad \quad (e^x)^2 \\
 & = \int_1^3 \frac{1}{u^2+3^2} du \\
 & = \left[\frac{1}{3} \tan^{-1} \left(\frac{u}{3} \right) \right]_1^3 \\
 & = \frac{1}{3} \left[\underbrace{\tan^{-1}(1) - \tan^{-1} \left(\frac{1}{3} \right)}_{\pi/4} \right] \\
 & = \frac{\pi}{12} - \frac{1}{3} \tan^{-1} \left(\frac{1}{3} \right)
 \end{aligned}$$



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Explanation

When changing the boundaries of a definite integral during an exponential substitution, you will often need to evaluate the mathematical constant e raised to the power of a natural logarithm.

The best tip for handling this situation is to remember that the exponential function and the natural logarithm are perfect opposites, meaning they completely undo each other. Whenever you encounter the constant e raised to the natural logarithm of a number, you can instantly strip away both operations and simply write down that original number.

This clever shortcut allows you to quickly simplify complicated logarithmic boundaries into clean, straightforward values, making it much easier to compute the final steps of your calculus problem.

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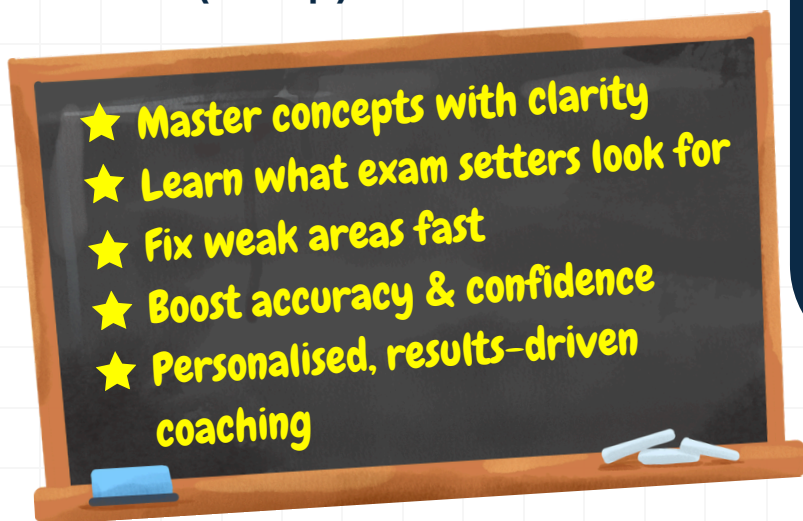
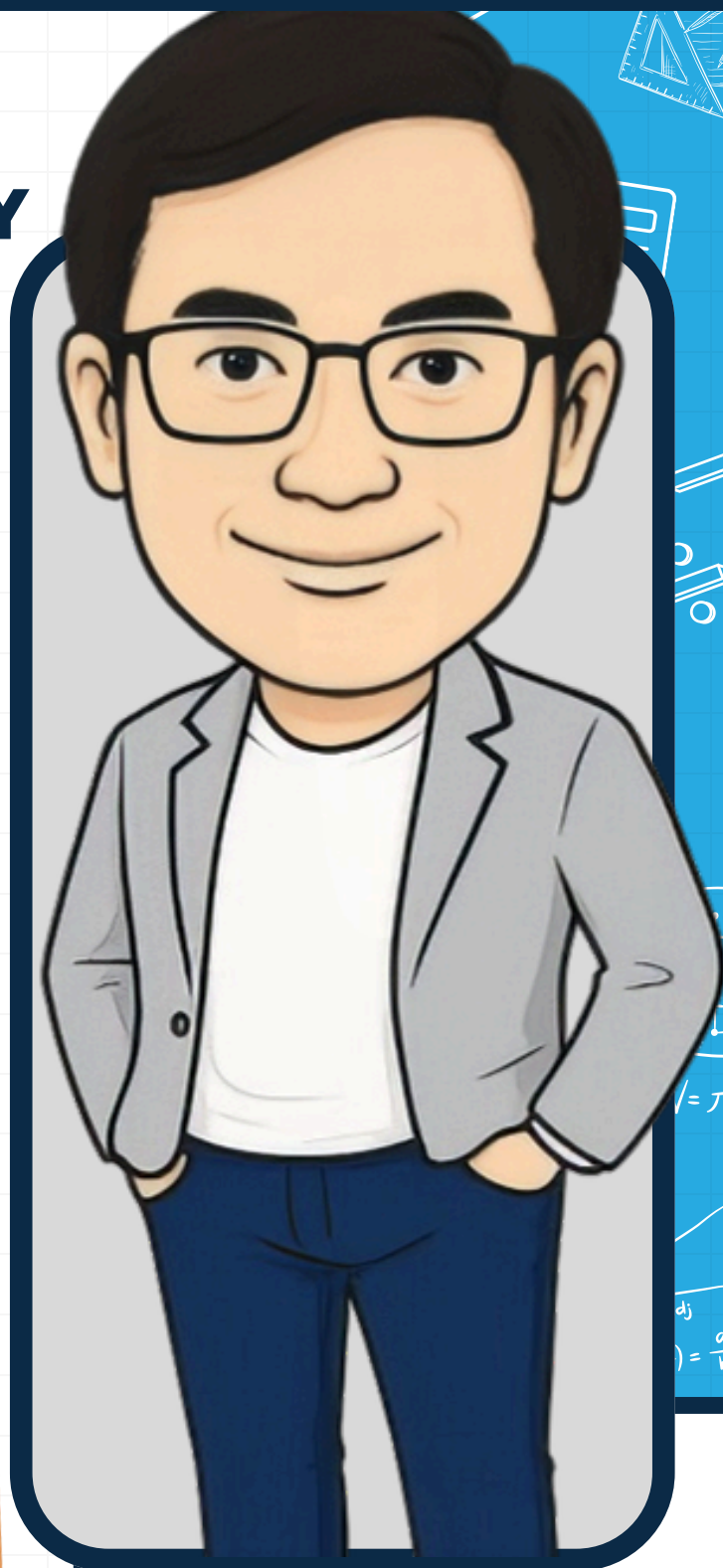
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